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Statistical Developments in Life Testing

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STATISTICAL DEVELOPMENTS IN LIFE TESTING[a]

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Summary

In this paper we describe recently developed statistical methods for analyzing data arising from life tests and for designing life tests. Advantage is taken of the time ordered nature of life test data to shorten substantially the time required to reach a decision. Most of the results ~~have been~~ obtained under the assumption of an exponential distribution of life. Replacement, non replacement, sequential, non sequential, and truncated procedures are described. Some useful tables are given, ~~[illegible]~~

It is a characteristic feature of most life and fatigue tests that they give rise to ordered observations. If, for example, twenty radio tubes are place on life test and t_i denotes the time when the ith tube fails, the data occur in such a way that $t_1 \leq t_2 \leq \cdots \leq t_{20}$. Exactly the same kind of ordered situation will occur whether the problem under consideration deals with the life of electric bulbs, the life of electronic components, the life of ball bearings, or the length of life of human beings after they are treated for a disease. The examples we have just given all involved ordering in time. This need not necessarily be the case. If we are interested in destructive test situations involving such things as the current needed to blow a fuse, the voltage needed to break down a condenser, the force needed to rupture a physical

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material, then we can often arrange to test in such a way that every item in the sample is subjected to precisely the same stimulus (current, voltage, stress). If this is done, then clearly the weakest item will be observed to fail first, the second weakest next, etc. In the present paper we discuss almost exclusively situations in which it is the time to failure that is the important random variable, and therefore we shall use the language of time throughout the paper. It should be emphasized, however, that there will be some practical problems which do not involve time, but for which some of the ideas discussed in this paper are quite relevant.

Put in general terms, we test n items drawn at random from some population and the data become available in such a way that the smallest observation comes first, the second smallest second, ..., and finally the largest observation last. Clearly we can, if we choose, discontinue experimentation long before all n items have failed. In particular we may decide to terminate the experiment as soon as we have the first r ($\leq n$) failures, or we may decide to stop at some preassigned truncation time T_0, or we may adopt a sequential procedure permitting us to stop as soon as certain conditions are met. In all of these cases our primary concern is the development of statistical procedures which, by taking advantage of the fact that data become available in order, will enable the experimenter to reach a decision in a shorter time or with fewer observations, than would be possible if data did not become available in a time ordered way.

II. <u>Preliminary remarks on the exponential distribution.</u>

In this paper virtually all results will be obtained under the assumption that the length of life X has an exponential distribution described by the probability density function (henceforth abbreviated as p. d. f.) $f(x|\theta)$ of the form

$$(1) \qquad f(x|\theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x \geq 0, \ \theta > 0$$
$$= 0, \text{ elsewhere.}$$

A partial justification for this assumption has been discussed in some detail by the author[1] and several relevant references are given in that paper. Quite recently further evidence of an empirical nature can be found in a series of ARINC monographs. We are well aware of the fact that many life distributions are not adequately described by equation (1). However, we feel that an understanding of the theory in the exponential case is essential if we are to treat more general situations. In fact, in some cases, the solution for a p. d. f. which is not of the form (1) can be readily obtained by making trivial modifications of the results in the exponential case. We intend to discuss this question in detail in another paper.

Returning then to the p. d. f. (1) we state some results which are discussed in detail in a paper by Epstein and Sobel[3]. The first result is as follows: Let n items be drawn at random from a distribution whose p. d. f. is given by (1) and placed on life test. Let the observations become available in order, i.e., $x_{1,n} \leq x_{2,n} \leq \ldots \leq x_{r,n} \leq \ldots \leq x_{n,n}$ where by $x_{i,n}$ is meant the time when the i'th failure occurs. Suppose that experimentation is discontinued as soon as the r'th item fails (r is preassigned), then it can be shown that the maximum likelihood estimate of the mean life[b] θ is given by $\hat{\theta}_{r,n}$ where

$$(2) \quad \hat{\theta}_{r,n} = \frac{x_{1,n} + x_{2,n} + \ldots + x_{r,n} + (n-r)\,x_{r,n}}{r}.$$

In words we add up the total number of hours lived by all items, those that failed and those which did not fail, and divide by the number of failures. The estimate $\hat{\theta}_{r,n}$ is

(b) θ is the mean life since $E(X) = \int_0^\infty x \frac{1}{\theta} e^{-x/\theta}\,dx = \theta$.

"best" in the sense that in addition to being maximum likelihood, it is also unbiased, minimum variance, efficient, and sufficient. The p. d. f. of $\hat{\theta}_{r,n}$ is given by

$$(3)\qquad f_r(y) = \frac{1}{(r-1)!}\,(r/\theta)^r\, y^{r-1}\, e^{-ry/\theta}\,,\quad y > 0$$
$$= 0,\ \text{elsewhere}$$

and $2r\hat{\theta}_{r,n}/\theta$ is distributed as chi-square with $2r$ degrees of freedom (which we denote as $\chi^2(2r)$).

In the preceding paragraph we have been concerned with the non-replacement situation where one does not replace failed items at once by new items drawn from the underlying p. d. f. (1). In the replacement case (where one immediately replaces a failed item by a new one) it can be shown that the maximum likelihood estimate of the mean life θ is given by

$$(4)\qquad \hat{\theta}_{r,n} = n\,x_{r,n}/r,$$

where by $x_{r,n}$ is meant the total time (measured from the beginning of the life test) to observe the r'th failure and where the sample size n is maintained throughout the life test. It should be remarked that $nx_{r,n}$ is the total number of hours lived by all items on test since

$$(5)\qquad nx_{r,n} = nx_{1,n} + n(x_{2,n} - x_{1,n}) + n(x_{3,n} - x_{2,n}) + \dots + n(x_{r,n} - x_{r-1,n}).$$

On the righthand side of (5), $nx_{1,n}$ is the number of hours lived by all items up to the time the first failure occurred, and $n(x_{i,n} - x_{i-1,n})$ is the number of hours lived by all items between the times of occurrence of the $(i-1)$st failure and i'th failure. The estimate (4) in the replacement case has precisely the same distribution and the same optimum properties as does the estimate (2) in the non-replacement case. In fact if we

Let $T_{r,n}$ be the total number of hours lived by all items whether they failed or not, up to the time when the r^{th} failure occurred, one can write both (2) and (4) as

(6) $\hat{\theta}_{r,n} = T_{r,n}/r$

where

$$T_{r,n} = x_{1,n} + x_{2,n} + \dots + x_{r-1,n} + (n - r + 1)\, x_{r,n}$$

in the non-replacement case and where

$$T_{r,n} = n x_{r,n}$$

in the replacement case. In either case, $2T_{r,n}/\theta$ is distributed as $\chi^2(2r)$.

An interesting and important feature of the distribution of $\hat{\theta}_{r,n}$ in either the replacement or non-replacement case is its independence of n. It therefore follows that no matter what n is a $100(1-\alpha)$ percent confidence interval for the true but unknown mean life θ based on a test terminated after one has observed the first r out of n failures is given by

$$(7)\quad \left(\frac{2r\hat{\theta}}{\chi^2_{\frac{\alpha}{2}}(2r)} , \frac{2r\hat{\theta}}{\chi^2_{1-\frac{\alpha}{2}}(2r)} \right) = \left(\frac{2T_r}{\chi^2_{\frac{\alpha}{2}}(2r)} , \frac{2T_r}{\chi^2_{1-\frac{\alpha}{2}}(2r)} \right)$$

where we define the constant $\chi^2_\gamma(2k)$ by the equation

$$(8)\quad \Pr\left(\chi^2(2k) > \chi^2_\gamma(2k)\right) = \gamma.$$

Similarly, suppose we want to find a test procedure which will give a prescribed operating characteristic curve (henceforth abbreviated as O. C. curve). Put in statistical terms(*) we want to test the hypothesis $H_0: \theta = \theta_0$ against the alternative H_1:

(*) θ_0 is some acceptable (good) mean life, θ_1 is some unacceptable (low) mean life, α is the producer's risk and β is the consumer's risk.

$\theta = \theta_1 < \theta_0$ subject to the conditions that for $\theta = \theta_0$, $L(\theta_0)$ = Pr(accepting $\theta = \theta_0 \mid \theta_0$ is true) = $1 - \alpha$ and for $\theta = \theta_1$, $L(\theta_1)$ = Pr (accepting $\theta = \theta_0 \mid \theta_1$ is true) $\leq \beta$. It is shown in our paper(1) that the region of acceptance for $\theta = \theta_0$ must be of the form

(9) $$\hat{\theta}_{r,n} > C = \theta_0 \chi^2_{1-\alpha}(2r)/2r,$$

where the O. C. curve based on this region of acceptance must be independent of n, since the distribution of $\hat{\theta}_{r,n}$ depends only on r. The appropriate values of r (and hence C) for certain values of α, β, and θ_0/θ_1 are given in Table 1. For values of α, β and θ_0/θ_1 not given in the table, the appropriate r to use is the smallest integer r such that $\chi^2_{1-\alpha}(2r)/\chi^2_{\beta}(2r) \geq \theta_1/\theta_0$.

In the test procedure $\hat{\theta}_{r,n} > C$, the sample size n is at our disposal. The effect of increasing n is to shorten the time needed on the average to reach a decision and thus if we happen to be in a situation where the items being tested are cheap but where time is very valuable, we may well prefer a test of the form $\hat{\theta}_{r,n} > C$ to one which is of the form $\hat{\theta}_{r,r} > C$. These two procedures have exactly the same O. C. curve and our only reason for preferring a rule of action based on the first r failures out of n items tested to one based on failing all r out of r items is that the first rule of action will take a shorter time on the average. Thus, for example, a test procedure which involves stopping an experiment after the first of two items on test has failed will lead to rules of action whose O. C. curve is precisely the same as that found by placing one item on test and waiting until it fails. However, the expected length of time in the first procedure is only one half that in the second procedure. Consequently, if the time saved outweighs the loss due to testing two items rather than one, we would prefer the first procedure.

Let $E(X_{r,n})$ be the expected length of time needed to observe the first r failures out of n items placed on test, and let $E(X_{r,r})$ be the expected length of time needed to

observe all r items to fail, if r items are placed on test, then the ratio

(10) $\phi_{r,n} = E(X_{r,n})/E(X_{r,r})$

is a measure of the expected saving in time due to using the first procedure as compared with the second procedure. In Table 2 we give the values of this ratio for selected small values of r and n, in the non-replacement case. This table shows, that if "time is money", procedures which terminate before the whole sample is observed may be very advantageous. In evaluating (10) the following formulae are useful:

(11) $$E(X_{r,n}) = \theta\left(\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{n-r+1}\right) = \theta \sum_{j=1}^{r} 1/(n-j+1),$$

in the case where failed items are not replaced and

(12) $E(X_{r,n}) = r\theta/n$

in the case where failed items are replaced at once by new items drawn from the p. d. f. (1)

III Truncated life tests

It is frequently necessary on practical grounds to terminate a life test by a preassigned time T_0. This leads to truncated tests in which it is decided in advance that the life test will be terminated at min $(X_{r_0,n};\ T_0)$ where $X_{r_0,n}$ is the time at which the r_0'th failure occurs and T_0 is the truncation time beyond which the life test will not be allowed to run. If the life test is terminated at $X_{r_0,n}$ (i.e., r_0 failures occur before time T_0) then the action taken will be to reject. If the experiment is terminated at time T_0 (i.e., the r_0'th occurs after time T_0) then the action in terms of "hypothesis" testing is acceptance. In a paper by Epstein[4] one can find details concerning such test procedures for both the replacement and non-replacement cases. These test procedures are characterized by three functions

$E_\theta(r)$, $E_\theta(T)$, and $L(\theta)$, the expected number of observations to reach a decision, the expected waiting time to reach a decision, and the probability of accepting respectively, if θ is the true value. The formulas are given below.

In the non-replacement case

$$(13) \quad E_\theta(r) = np_\theta \left[\sum_{k=0}^{r_0-2} b(k|n-1,p_\theta) \right] + r_0 \left[1 - \sum_{k=0}^{r_0-1} b(k|n,p_\theta) \right]$$

where

$$p_\theta = 1 - e^{-T_0/\theta} \quad \text{and} \quad b(k|n,p_\theta) = \binom{n}{k} p_\theta^{\,k} (1-p_\theta)^{n-k}.$$

The probability distribution of r is given by

$$(14) \quad Pr(r = k \mid \theta) = b(k|n,p_\theta), \; k = 0, 1, 2, \ldots, r_0 - 1$$

and

$$(14') \quad Pr(r = r_0 \mid \theta) = 1 - \sum_{k=0}^{r_0-1} Pr(r = k \mid \theta).$$

Further one has

$$(15) \quad E_\theta(T) = \sum_{k=1}^{r_0} Pr(r = k \mid \theta) \; E_\theta(X_{k,n})$$

where $E_\theta(X_{k,n})$ can be found from (11), and

$$(16) \quad L(\theta) = \sum_{k=0}^{r_0-1} Pr(r = k \mid \theta).$$

In the replacement case the probability distribution of r is given by

(17) $\Pr(r = k \mid \theta) = p(k; \lambda_\theta), \quad k = 0, 1, 2, \ldots, r_0 - 1$

and

(17') $\Pr(r = r_0 \mid \theta) = 1 - \sum_{k=0}^{r_0-1} p(k; \lambda_\theta)$

In (17) and (17'), $\lambda_\theta = nT_0/\theta$ and $p(k; \lambda_\theta) = \lambda_\theta^k \exp(-\lambda_\theta)/k!$

Further one has

(18) $E_\theta(r) = \lambda_\theta \sum_{k=0}^{r_0-2} p(k; \lambda_\theta) + r_0\left[1 - \sum_{k=0}^{r_0-1} p(k; \lambda_\theta)\right]$

(19) $E_\theta(T) = \theta E_\theta(r)/n$

and

(20) $L(\theta) = \sum_{k=0}^{r_0-1} p(k; \lambda_\theta)$

We have just given formulas for the O. C. curve, the expected waiting time, and expected number of items failed in the course of reaching a decision for any preassigned n, T_0, r_0. We now give a formula for finding the appropriate truncated test (that is, for finding r_0 and n) when the truncation time T_0 is preassigned and the O. C. curve is required (for preassigned type I error, α, and type II error, β) to be such that $L(\theta_0) = 1 - \alpha$ and $L(\theta_1) \leq \beta$. It is proved in the paper referred to in the first paragraph of this section that for both the replacement case and the non-replacement case the appropriate r_0 is precisely the same as the r_0 used in tests of the form (9). Hence Table 1 can be used. As for the appropriate value of n one should choose

$$n = \left[\theta_0\, \chi^2_{1-\alpha}(2r_0)/2T_0\right] \tag{21}$$

where $[x]$ means the greatest integer $\leq x$, in the replacement case.

In the non-replacement situation a good approximate value of n, in case θ_0/T_0 is substantially more than one (say ≥ 3), is given by

$$n = \left[r_0/(1 - e^{-T_0/C})\right] \tag{22}$$

where $C = \theta_0\, \chi^2_{1-\alpha}(2r_0)/2r_0$.

IV. Sequential Life Tests

One can make substantial improvements on the procedures described in sections II and III by following a sequential procedure. It is shown in a paper by Epstein and Sobel[5] that the sequential probability ratio test of A. Wald can be applied in life testing. It is very interesting that decisions can now be made continuously in time. At each moment t, one can decide either to accept, to reject, or to continue the life test. If we are, as before, testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ $(\theta_0 > \theta_1)$ with Type I error $= \alpha$ and Type II error $= \beta$, then the decision as time unfolds depends on

$$B < (\theta_0/\theta_1)^r \exp - \left\{(1/\theta_1 - 1/\theta_0)V(t)\right\} < A \tag{23}$$

where A and B can for all practical purposes be taken as

$$A = (1 - \beta)/\alpha \quad \text{and} \quad B = \beta/(1 - \alpha) \tag{24}$$

In (23), r is the number of failures observed by time t. The decision to continue experimentation is made as long as the inequality (23) holds. As soon as (23) is violated, one accepts H_0 if the function of t in (23) is $\leq B$, and one rejects H_0 (accepts H_1) if the function of t in (23) is $\geq A$.

In (23) $V(t)$ is a statistic which equals the total number of hours lived by all items, failed [illegible] up to time t. In the replacement case

(25) $$V(t) = nt,$$

while in the nonreplacement case[4]

(26) $$V(t) = \sum_{i=1}^{r} (n - i + 1)(x_i - x_{i-1}) + (n - r)(t - x_r) = \sum_{i=1}^{r} x_i + (n - r)(t - x_r).$$

It is convenient to write (21) as

(27) $$-h_1 + rs < V(t) < h_0 + rs,$$

where h_0, h_1, and s are positive constants given by

(28) $$h_0 = \frac{-\log B}{1/\theta_1 - 1/\theta_0}, \quad h_1 = \frac{\log A}{1/\theta_1 - 1/\theta_0}, \quad s = \frac{\log(\theta_0/\theta_1)}{1/\theta_1 - 1/\theta_0}.$$

It is shown in our paper referred to in the first paragraph how formula (27) enables one to carry out the sequential procedure graphically.

The O. C. curve, that is, the probability of accepting H_0 when θ is the true parameter value, is given approximately by a pair of parametric equations

(29) $$L(\theta) = \frac{A^h - 1}{A^h - B^h}, \quad \theta = \frac{(\theta_0/\theta_1)^h - 1}{h(1/\theta_1 - 1/\theta_0)},$$

by letting the parameter h run through all real values.

The values of $L(\theta)$ at the five points $\theta = 0, \theta_1, s, \theta_0, \infty$ enable one to sketch the entire curve. These values are respectively 0, β, $\log A/(\log A - \log B)$, $1 - \alpha$, and 1.

$E_\theta(r)$, the expected number of observations required to reach a decision, when θ is the mean life is given by

(30) $$E_\theta(r) \sim \begin{cases} \dfrac{h_1 - L(\theta)(h_0 + h_1)}{s - \theta}, & \theta \neq s \\[2ex] \dfrac{h_0 h_1}{s^2}, & \theta = s \end{cases}$$

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[4] It should be remarked that in the non-replacement case a special problem arises if all n items fail without reaching a decision. This eventuality can be taken care of in ...

If we let $k = \theta_1/\theta_0$, the approximate values of $E_\theta(r)$ become particularly simple when $\theta = \theta_1$, s, or θ_0. They are

(31) $$E_{\theta_1}(r) \sim \left[\beta \log B + (1-\beta)\log A\right] / \left[\log k - (k-1)/k\right]$$

$$E_s(r) \sim -\log A \log B / (\log k)^2 ,$$

$$E_{\theta_0}(r) \sim \left[(1-\alpha)\log B + \alpha \log A\right] / \left[\log k - (k-1)\right].$$

In Table 3, we give $E_\theta(r)$ for five values of θ $(0, \theta_1, s, \theta_0, \infty)$ for four values of k $(1/2, 2, 3/2, 3)$, and for the four number pairs (α, β) which can be made with the numbers .01 and .05.

It can be shown that $E_\theta(t)$, the expected waiting time to reach a decision is given by the formula

(32) $$E_\theta(t) = E_\theta(r)\,\theta/n$$

in the replacement case. In the non-replacement case,

(33) $$E_\theta(t) = \sum_{k=1}^{n} Pr(r = k \mid \theta)\; E_\theta(X_{k,n})$$

where $E_\theta(X_{k,n})$ can be found from (11). A good approximation for $E_\theta(t)$ is given by

(34) $$E_\theta(t) \sim \theta \log\left(\frac{n}{n - E_\theta(r)}\right).$$

The derivations of all formulas in this section can be found in the reference cited in the first paragraph.

V. Conclusion

We have not attempted in this paper to cover all of the papers which have been published by a number of writers including the author in the field of life testing. We have selected essentially three papers [3,4,5] which give some of the results which we consider to be most fundamental. A careful reading of these papers gives a good introduction to the statistical methodology involved in life testing. These papers also contain many numerical illustrations which will be of substantial help in seeing how one applies the theory to the design and analysis of life tests.

References

(1) Benjamin Epstein, "Statistical Problems in Life Testing", Proceedings of the Seventh Annual Convention of the American Society for Quality Control, pp. 385 – 398, 1953.

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(3) B. Epstein and M. Sobel, "Life Testing", Journal of the American Statistical Association 48, 486 – 502, 1953.

(4) B. Epstein, "Truncated Life Tests in the Exponential Case", Annals of Mathematical Statistics 25, 555 – 564, 1954.

(5) B. Epstein and M. Sobel, "Sequential Life Tests in the Exponential Case", Annals of Mathematical Statistics 26, 82 – 93, 1955.

Table 1

Values of r (upper numbers) and of $\chi^2_{1-\alpha}(2r)/2$ (lower numbers) such that the test based on using $\hat{\theta}_{r,n} > C = \theta_0 \chi^2_{1-\alpha}(2r)/2r$ as an acceptance region for $\theta = \theta_0$ will have

$L(\theta_0) = 1 - \alpha$ and $L(\theta_1) = \beta$.

θ_0/θ_1	$\alpha = .01$			$\alpha = .05$			$\alpha = .10$		
	$\beta = .01$	$\beta = .05$	$\beta = .10$	$\beta = .01$	$\beta = .05$	$\beta = .10$	$\beta = .01$	$\beta = .05$	$\beta = .10$
3/2	136 110.4	101 79.1	83 63.1	95 79.6	67 56.1	55 43.6	77 66.0	52 43.0	41 33.0
2	46 31.7	35 22.7	30 18.7	33 [illegible]	24 15.7	19 12.4	26 17.7	18 12.8	15 10.3
5/2	27 18.4	21 11.8	18 9.62	19 12.4	14 8.46	11 6.17	15 10.[illegible]	11 7.02	9 5.43
3	19 10.1	15 7.48	13 6.10	13 7.67	10 5.43	8 3.98	11 7.02	8 4.88	6 3.13
4	12 5.43	10 4.13	9 3.51	9 [illegible]	7 3.79	6 2.61	7 [illegible]	5 2.61	6 1.75
5	9 3.41	8 2.91	7 2.19	7 [illegible]	5 [illegible]	4 1.17	5 [illegible]	4 1.77	3 1.10
10	3 1.28	4 .983	4 .823	4 1.37	3 .818	3 .818	3 1.10	2 .330	2 .532

(14)

Table 2

Ratio of the Expected Waiting Time to Observe the r'th Failure in Samples of Size n and r respectively.

$$E(X_{r,n}) / E(X_{r,r}) = \alpha_{r,n}$$

r \ n	1	2	3	4	5	10	15	20
1	1	[illegible]	[illegible]	[illegible]	.20	[illegible]	.0[illegible]7	.050
2	-	1	.56	.39	.30	.14	.092	.068
3	-	-	1	.59	.43	.18	.12	.087
4	-	-	-	1	.62	.23	.14	.104
5	-	-	-	-	1	.28	.18	.125
10	-	-	-	-	-	1	.35	.23

Table 3

Approximate values of $\bar{n}_{\theta}(r)$ for sequential tests for various values of $k = \theta_0/\theta_1$ and α, β.

$k = \theta_0/\theta_1$		3/2		[illegible]		5/2		3	
	α	.01	.05	.01	.05	.01	.05	.01	.05
θ	β								
0	.01	11	7	7	4	5	3	4	3
	.05	11	7	7	4	5	3	4	3
θ_1	.01	62.4	40.1	25.1	15.1	14.2	9.20	10.4	6.74
	.05	60.4	36.7	22.6	13.7	13.8	8.38	10.1	6.14
[illegible]	.01	12[illegible]	82.7	43.9	20.3	[illegible]	16.2	17.5	11.1
	.05	82.7	52.7	28.3	18.0	[illegible]	[illegible]	[illegible]	7.38
[illegible]	.01	47.6	44.9	14.7	13.6	[illegible]	7.16	[illegible]	[illegible]
	.05	10.6	25.0	9.40	6.06	[illegible]	[illegible]	[illegible]	[illegible]
∞	any	0	0	0	0	0	0	0	0